SELF-SIMILAR SOLUTIONS OF THE EXTERNAL CONDENSATION PROBLEM

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A new class of self-similar solutions of the external condensation problem is constructed, which takes account of the change in the process parameters along the length of the condensation section.

Known self-similar solutions of the external condensation problem permit the computation of the vapor condensation process on a flat vertical impermeable surface during motion of the condensate film under the effect of gravitation [1, 2], or on a horizontal flat impermeable surface during entrainment of the film by a moving vapor stream [3, 4]. Intensification of the condensation process is achieved by suction of the condensate film through a solid porous wall, by blowing the film with a vapor stream, by using condensers with a complex surface shape. Construction of the self-similar solutions taking account of the combined influence of the external mass force field, which is variable along the length of the condensation section (during condensation on curved surfaces), the inhomogeneous tangential stress, the pressure gradient, and blowing (suction) of the condensate through a solid wall considerably broadens the domain of practical application of the self-similar methods.

The system of equations describing the film condensation process for saturation vapor on a curvilinear surface in the presence of fluid blowing (suction) through a solid wall and tangential stress on the vapor — liquid interface, has the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0; \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=f_0(x)+\nu\frac{\partial^2 u}{\partial y^2}; \quad (2)$$

$$u\frac{\partial (T_1-T_s)}{\partial x}+v\frac{\partial (T_1-T_s)}{\partial y}=a\frac{\partial^2 (T_1-T_s)}{\partial y^2}; \quad (3)$$

$$y=0:\quad u=0;\quad v=f_1(x);\quad T_1-T_s=f_2(x); \quad (4)$$

$$y=\delta:\quad \frac{\partial u}{\partial y}=f_3(x);\quad T_1-T_s=0;$$

$$\rho h\left(u\frac{\partial \delta}{\partial x}+v\right)=k\frac{\partial^2 (T_1-T_s)}{\partial y^2}, \quad (5)$$

where $f_0(x)=-(1/\rho)(\partial P/\partial x)+g\cos\alpha$. Introducing the stream function ψ, $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$, according to (1), we rewrite (2)-(5) as follows

$$\begin{gathered}\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\nu\frac{\partial^3\psi}{\partial y^3}+f_0(x);\\ \frac{\partial\psi}{\partial y}\frac{\partial (T_1-T_s)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial (T_1-T_s)}{\partial y}=a\frac{\partial^2 (T_1-T_s)}{\partial y^2};\\ y=0:\ \frac{\partial\psi}{\partial x}=-f_1(x);\quad \frac{\partial\psi}{\partial y}=0;\quad T_1-T_s=f_2(x);\\ y=\delta:\ \frac{\partial^2\psi}{\partial y^2}=f_3(x);\quad T_1-T_s=0;\\ \rho h\left(\frac{\partial\psi}{\partial y}\frac{\partial\delta}{\partial x}-\frac{\partial\psi}{\partial x}\right)=k\frac{\partial (T_1-T_s)}{\partial y}.\end{gathered} \quad (6)$$

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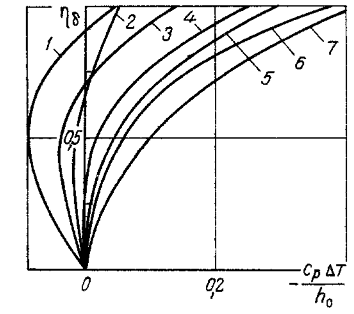

Fig. 1

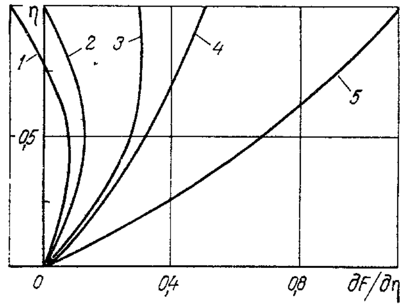

Fig. 2

Fig. 1. Dependence of the dimensionless condensate film thickness on the solid wall curvature, the blowing (suction) intensity of the liquid through the wall, and the tangential stress on the interphasal surface: 1) $C_1 = -0.2$; $C_0 = C_3 = 0$; 2) $C_3 = -0.5$; $C_0 = C_1 = 0$; 3) $C_1 = -0.1$; $C_0 = C_3 = 0$; 4) $C_0 = C_1 = C_3 = 0$; 5) $C_3 = 0.2$; $C_0 = C_1 = 0$; 6) $C_0 = 2$; $C_1 = C_3 = 0$; 7) $C_1 = 0.2$; $C_0 = C_3 = 0$.

Fig. 2. Dimensionless velocity of the condensate film in the presence of a tangential stress on the interphasal surface: 1) $C_3 = -0.6$; 2) $C_3 = -0.5$; 3) $C_3 = -0.2$; 4) $C_3 = 0$; 5) $C_3 = 0.6$.

System (6) is written in the dimensionless variables $\eta = y\nu^{-0.5} x^{-0.25} f_0^{0.25}$, $F(\eta) = \psi \nu^{-0.5} x^{-0.75} f_0^{-0.25}$, $\Theta(\eta) = (T_1 - T_s)/f_2(x)$ as:

$$F''' + FF''\left(\frac{3}{4} + \frac{1}{4}\,\frac{x}{f_0}\,\frac{\partial f_0}{\partial x}\right) - (F')^2\left(\frac{1}{2} + \frac{1}{2}\,\frac{x}{f_0}\,\frac{\partial f_0}{\partial x}\right) + 1 = 0; \tag{7}$$

$$\Theta''\frac{a}{\nu} + \Theta' F\left(\frac{3}{4} + \frac{1}{4}\,\frac{x}{f_0}\,\frac{\partial f_0}{\partial x}\right) + \Theta F'\,\frac{\partial f_2}{\partial x}\,\frac{x}{f_2} = 0;$$

$$\eta = 0:\quad F\left(\frac{3}{4} + \frac{1}{4}\,\frac{x}{f_0}\,\frac{\partial f_0}{\partial x}\right) = f_1 \nu^{-0.5} x^{0.25} f_0^{-0.25};\ F' = 0;\ \Theta = 1;$$

$$\eta = \eta_\delta:\quad F'' = f_3 \nu^{0.5} x^{-0.25} f_0^{-0.75};\quad \Theta = 0;$$

$$F\left(\frac{3}{4} + \frac{1}{4}\,\frac{x}{f_0}\,\frac{\partial f_0}{\partial x}\right) = \Theta'\,\frac{k(T_W - T_s)}{\rho h \nu}\,. \tag{8}$$

For self-similar solutions to exist it is necessary that the functions f_i have the following form:

$$f_0 = Dx^{C_0};\ f_1 = -C_1 \nu^{-0.5} x^{-0.25} f_0^{0.25};\ f_2 = C_2 = T_W - T_s = \Delta T = \text{const};$$

$$f_3 = C_3 \nu^{-0.5} x^{0.25} f_0^{0.75}. \tag{9}$$

The constants C_i are: C_0, the surface curvature and the pressure gradient; C_1, the blowing ($C_1 > 0$) or suction ($C_1 < 0$) intensity of the fluid through the solid wall; C_3, the magnitude of the tangential stress on the liquid–vapor interphasal surface.

Condensation occurs on the upper bulkhead of liquid propellant tanks during pressurization by a vapor and the condensate film hence moves under the effect of gravitation and the tangential stress caused by the injected vapor. The projections of the gravity force on the generator of the ellipsoidal bulkhead of the tank and the distribution of the tangential stress on the vapor – liquid interphasal surface are approximated by relationships (9) with a good degree of accuracy when the spraying atomizers are centrally arranged. The distributions of the gravitational and tangential stress forces are also approximated by the dependences of f_i during condensation of a moving vapor on the walls of converging tubes and nozzles. Suction of the condensate is used to diminish the fraction of the liquid phase in the vapor stream.

System (7)-(8) was solved by numerical integration and successive approximations. The difficulty in the numerical solution of boundary-value problem (7) is that the position of the boundary η_δ is not known and should be determined from the additional condition (8). Finding the dimensionless film thickness η_δ is performed by a simple gradient method from the condition

$$I = (F''(\eta_\delta) - C_3)^2 + \Theta^2(\eta_\delta) + \left[F(\eta_\delta)\left(\frac{3}{4} + \frac{1}{4}C_0\right) - \Theta'(\eta_\delta)\frac{kC_2}{\rho h \nu}\right]^2 = \min$$

in the following sequence. The initial approximations $F_1''(0)$, $\Theta_1'(0)$, $\eta_{\delta 1}$ are given in the first stage of the iteration process and system (7) is integrated numerically as a system with initial data. Projections of the gradient of the quality function I are found in the space $F''(0)$, $\Theta'(0)$, η_δ by a difference method. A step is taken in the direction of the antigradient of the quality function, and refined values of $F_2''(0)$, $\Theta_2'(0)$, $\eta_{\delta 2}$ are found. Then the procedure is repeated. The calculations are cut off under the condition $I < 10^{-4}$.

The method of directed search used for η_δ affords a savings of machine time as compared with the method in [1].

The first approximation system of the successive approximations has the form

$$F_0''' + 1 = 0;\quad \Theta_0'' = 0;\quad \eta = 0:\quad F' = 0;\quad F\left(\frac{3}{4} + \frac{1}{4}C_0\right) = C_1;\ \Theta = 1;$$
$$\eta = \eta_\delta:\quad F'' = C_3;\quad \Theta = 0. \tag{10}$$

The second approximation system is written thus:

$$F_1''' + 1 = -\left[F_0'' F_0\left(\frac{3}{4} + \frac{1}{4}C_0\right) - (F_0')^2\frac{1}{2}(1 + C_0)\right];$$
$$\Theta_1'' = -\Theta_0' F_0 \frac{\nu}{a}\left(\frac{3}{4} + \frac{1}{4}C_0\right);$$
$$\eta = 0:\quad F' = 0;\quad F_1\left(\frac{3}{4} + \frac{1}{4}C_0\right) = C_1;\quad \Theta = 1;$$
$$\eta = \eta_\delta:\quad F'' = C_3;\quad \Theta = 0. \tag{11}$$

As a result of solving system (10), (11) for the stream function and the temperature in the liquid film, we obtain the following dependences:

$$F_1 = \frac{4}{3 + C_0}\left[-\frac{\eta^7}{1260} + \frac{\eta^6}{180}(C_3 + \eta_\delta) - (C_3 + \eta_\delta)^2\frac{\eta^5}{120} + \frac{4C_1\eta^4}{24(3 + C_0)} - \frac{4C_1(C_3 + \eta_\delta)}{(3 + C_0)}\frac{\eta^3}{6}\right] -$$
$$-\frac{1}{2}(1 + C_0)\left[\frac{\eta^7}{720} - \frac{\eta^6}{120}(C_3 + \eta_\delta) + \frac{\eta^5}{60}(C_3 + \eta_\delta)^2\right] - \frac{\eta^3}{6}$$
$$+\frac{\eta^2}{2}\left[C_3 - \frac{4}{3 + C_0}\left(-\frac{\eta_\delta^5}{30} - \frac{C_3\eta_\delta^4}{6} - \frac{C_3^2 h_\delta^3}{6} -\right.\right.$$
$$\left.\left. -\frac{2C_1}{3 + C_0}\eta_\delta^2 - \frac{4C_1C_3\eta_\delta}{3 + C_0}\right)\right] + \frac{1}{2}(1 + C_0)\left[\frac{1}{6}\eta_\delta^5 + \frac{5}{12}C_3\eta_\delta^4 + \frac{C_3^2}{3}\eta_\delta^3 + \eta_\delta\right] + \frac{4C_1}{3 + C_0};$$
$$\Theta_1 = \frac{\nu}{a}\frac{3 + C_0}{4}\frac{1}{\eta_\delta}\left[-\frac{\eta^5}{120} + (C_3 + \eta_\delta)\frac{\eta^4}{24} + \frac{2C_1}{3 + C_0}\eta^2\right] -$$
$$-\eta\left[\frac{1}{\eta_\delta} + \frac{\nu}{a}\frac{3 + C_0}{4}\frac{1}{\eta_\delta^2}\left(-\frac{\eta_\delta^5}{120} + (C_3 + \eta_\delta)\frac{\eta_\delta^4}{24} + \frac{2C_1 + \eta_\delta^2}{3 + C_0}\right)\right] + 1. \tag{12}$$

Substituting (12) into (8), we find a relationship for the dimensionless film thickness η_δ. Higher approximations are constructed analogously.

The liquid discharge in the condensate film is determined from the dependence

$$G = \int_0^\delta \rho u dy = F(\eta_\delta)\nu^{0.5}x^{0.75}f_0^{0.25}\rho,$$

the friction on the solid wall equals

$$\tau = \mu\frac{\partial u}{\partial y} = F''(\eta)\nu^{-0.5}x^{0.25}f_0^{0.75}\mu,$$

and the heat flux in the solid wall is found by means of the relationship

$$q = -\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\lambda\Theta'(0)\nu^{0.5}x^{-0.25}f_0^{0.25}\Delta T.$$

The first approximation system (10) does not take account of the convective terms in the momentum and energy equations of the condensate film. Comparing the results of a successive approximation computation with the exact numerical solution shows that for fluids whose Prandtl number is $Pr = \nu/a > 10$, finding F and Θ by means of just the first approximation yields a relative error in the determination of the local heat transfer characteristics which is not greater than 2% in the range $0 < C_p\Delta T/h \leq 2$. For liquids whose Prandtl number is ≥ 1, the relative error does not exceed 6% in the same range of $C_p\Delta T/h$. The greatest deviation from the exact value is realized at $C_p\Delta T/h = 2$. Taking account of convective terms in the second approximation diminishes the relative error to 0.8% for liquids with $Pr \geq 1$. The relative error can reach 12-16% when using the first two approximations in a computation of the condensation of liquid metals ($Pr \sim 10^{-2}$-10^{-3}). The relative error of the third approximation does not exceed 5% in this case. Despite the awkwardness of the third-approximation expressions, successive approximation is considerably simpler and requires less time for realization than the numerical solution of the boundary-value problem (7).

The surface shape is determined by the quantity C_0. The film thickness increases along the length of the condensation section during condensation on surfaces for which $C_0 < 1$.

The film thickness is zero at the initial point (x=0). In the case $C_0 > 1$, an increase in the condensate discharge along the length of the condensation section is accompanied by a diminution in the film thickness, which is a result of the abrupt rise in the fluid velocity downstream in the film. In this case the formation and separation of liquid drops, caused by the Taylor instability, should be expected at the beginning of the condensation section. For $C_0 = 1$ the film thickness remains constant along the whole extent of the condensation section. The influence of wall curvature, blowing (suction) of the liquid through the wall, and surface friction on the film thickness is seen from Fig. 1.

In the presence of blowing ($C_1 > 0$), the conductive heat flux in the film on the vapor — liquid interphasal surface exceeds the heat flux in the solid wall, which is caused by accumulation of heat by the inflowing fluid. The reverse dependence is observed for suction of the condensate through the solid wall ($C_1 < 0$); i.e., the temperature gradient near the solid wall exceeds the temperature gradient on the interphasal surface. Coincidence of the directions of external mass force field and tangential stress actions on the liquid — vapor interphasal surface ($C_3 > 0$) accelerates the film flow; the velocity profile hence approaches the linear. When the directions of the external mass force and the tangential stress fields are opposite, the film flow is retarded, the heat flux is diminished, and the velocity on the surface of the liquid film (for $C_3 < 0$) may take a direction opposite to the velocity of the main condensate stream (Fig. 2).

NOTATION

x, y, coordinate axes directed along and normal to the solid wall; u, v, components of the velocity vector along the x and y axes; T_S, T_W, vapor saturation and solid wall temperatures; P, pressure; g, acceleration of gravity; ν, μ, kinetic and dynamic viscosities; ρ, condensate density; ψ, stream function; δ, condensate layer thickness; a, thermal diffusivity coefficient; k, heat conductivity coefficient; h, latent heat of condensation; τ, tangential stress; q, heat flux; C_i, constants; α, slope; F, dimensionless dependent variable; η, variable of the similarity transformation; Θ, dimensionless temperature; I, quality function.

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